CSCI 3210: Computational Game Theory

Intro to Computational Social Choice (COMSOC) and Voting

Handbook of COMSOC

Ch 1, 2

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http://mtirfan.com/CSCI-3210

Social choice theory

Individual preferences → collective choice

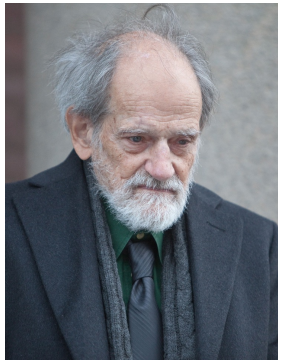
Jean-Charles de Borda
1770

Marquis de Condorcet
1785

Pierre-Simon Laplace
1795

Charles Dodgson
(Lewis Carroll)
1876

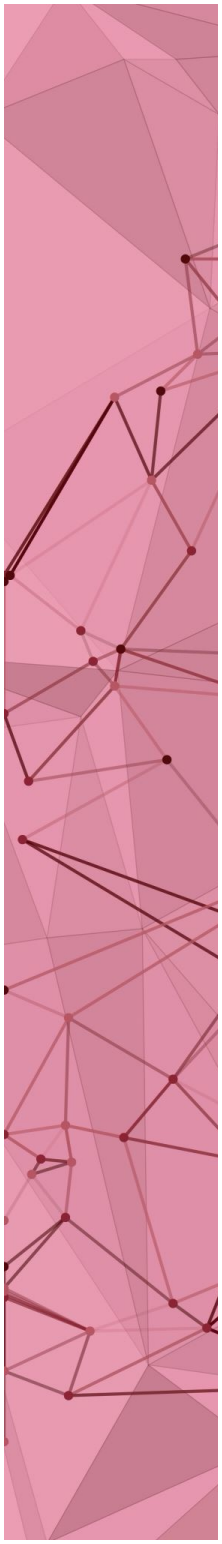
Kenneth Arrow
1951



Lloyd Shapley
1962

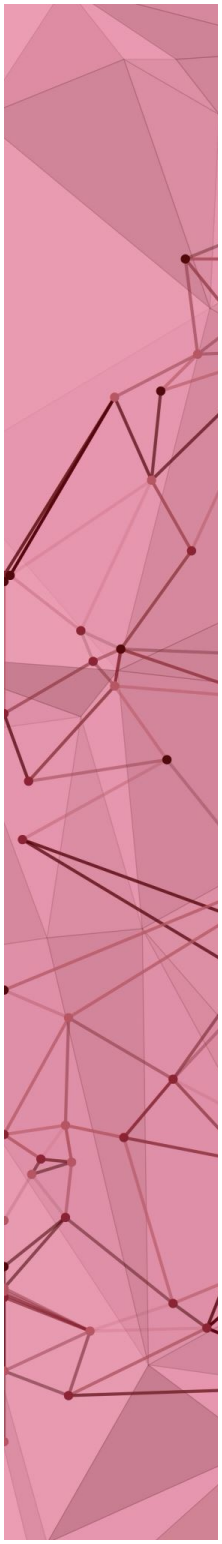
COMSOC Handbook:

“It is this interdisciplinary view [CS and social choice theory] on collective decision making that defines computational social choice as a field.”



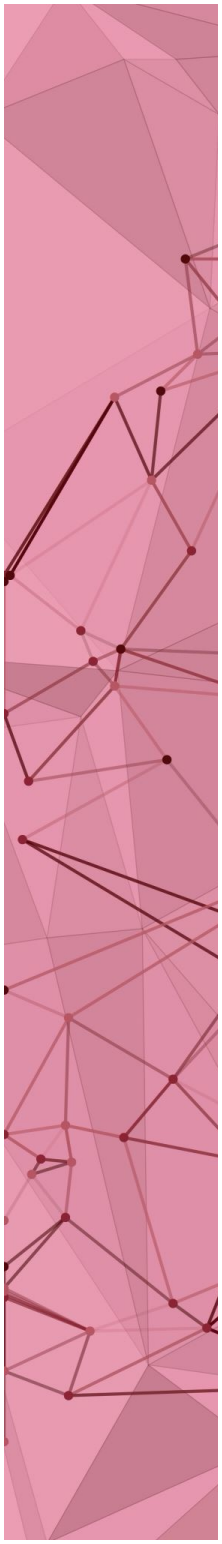
Research on COMSOC

- CS -> social choice theory
  - Design and analysis of algorithms for classical/preexisting SOC problems
  - Revisit old problems from scratch
  - Revival of social choice theory



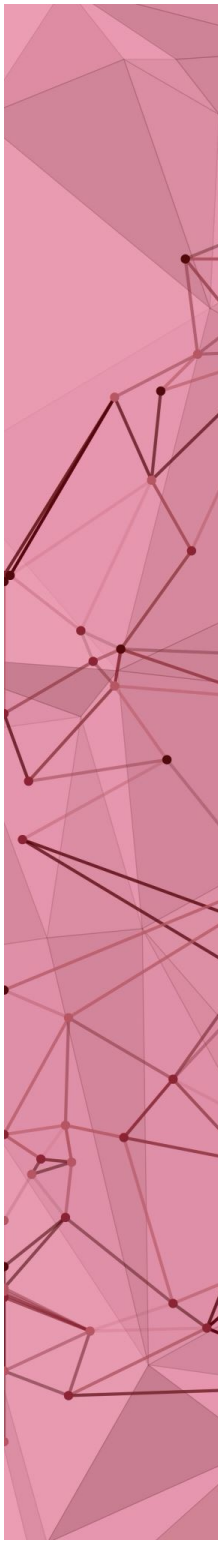
Research on COMSOC

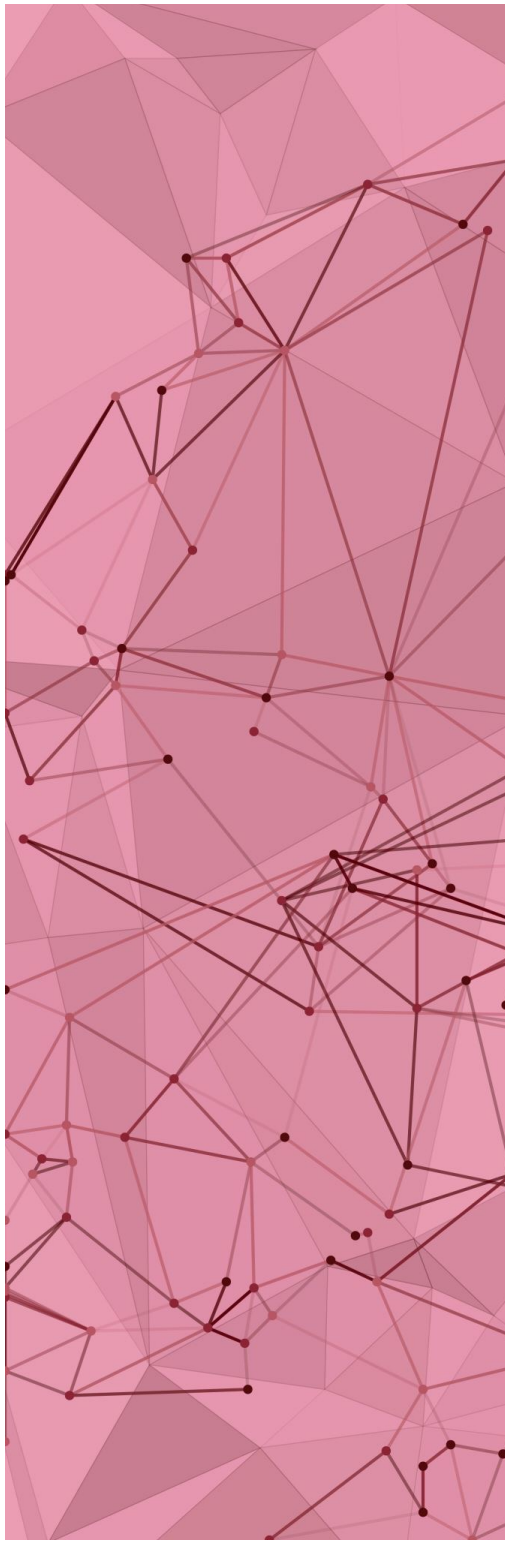
- Social choice theory -> CS applications
  - Recommendation for a group – restaurant/vacation
  - Information retrieval – aggregating information
  - Crowdsourcing – Mechanical Turk



Examples

- Elections
  - Social choice: winner
- Markets
  - Social choice: re-allocation of goods and money
- Auctions
  - Social choice: winner
- EPA regulations
  - Social choice: made by the government



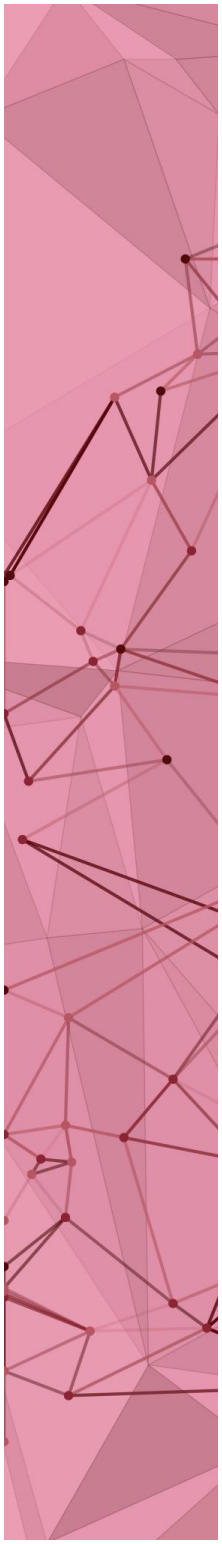


Voting

Ch 2 of Computational Soc Choice

First example: plurality vote

- Each voter marks one candidate
- The candidate with most votes wins
- Other names of plurality:
  - First-past-the-post
  - Choose-one
  - First-preference plurality



Example: plurality vote (vs. majority)

2025 New York City mayoral election

← 2021	November 4, 2025	2029 →
Registered	5,103,941[2]	
Turnout	2,218,647[1]	
	43.47% (▲20.08 pp)	

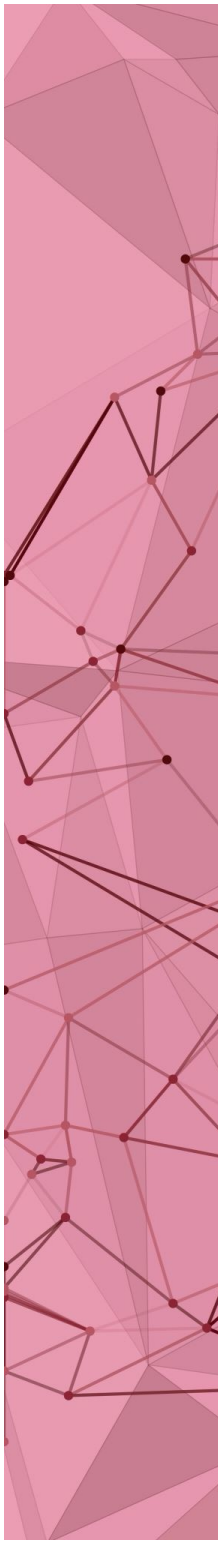


Nominee	**Zohran Mamdani**	Andrew Cuomo	Curtis Sliwa
Party	Democratic	Independent[a]	Republican[b]
Alliance	Working Families		
Popular vote	**1,114,184**	906,614	153,749
Percentage	**50.78%**	41.32%	7.01%

Got 50%+, but it was not needed under plurality

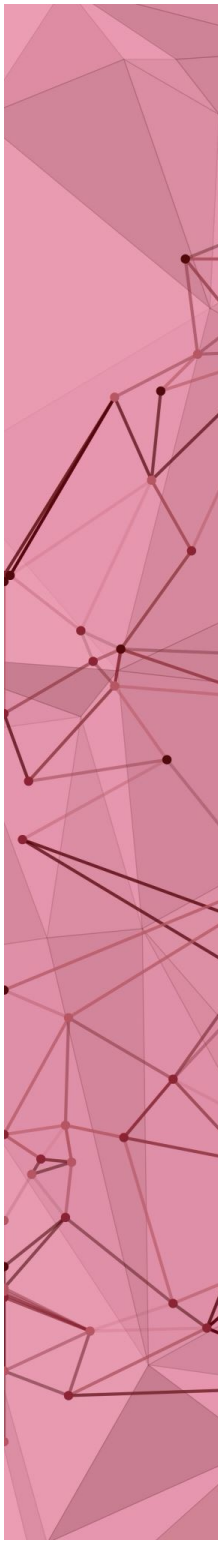
Critique of plurality

- Winner can be vastly unpopular (may get well below 50% votes) – numerous examples
- Vote splitting by a "spoiler" (candidate who loses at the end) – numerous examples
- Susceptible to *election control by deleting candidates*
  - Roman Senator Pliny the Younger's note (year 105)
  - Prisoners could be (A) acquitted, (B) banished, or (C) condemned to death
  - A had most support in Senate, but deletion of C led to B as the outcome



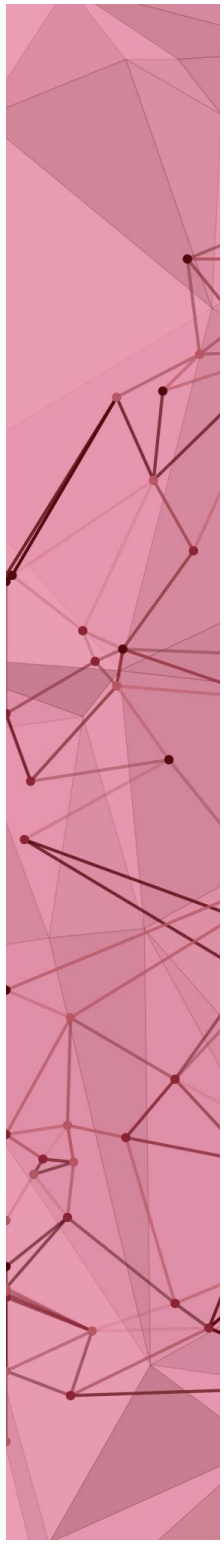
Ranked ballots

- Each voter ranks the candidates
- How to aggregate the rankings?
- Single round
  - Plurality: most frequent top choice candidate wins
  - Condorcet, Copeland, Borda, …
- Multiple rounds
  - Ranked-choice voting and variants



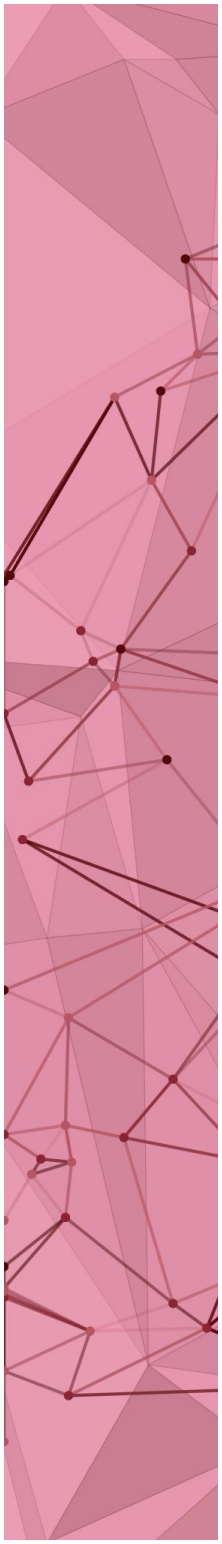
Formalization:
Social choice function (SCF)

- *Ballot of a voter:* linear ordering of candidates (no ties allowed)
- *Profile:* consists of a ballot for each voter
- *SCF:* Profile → Winner(s)



Social welfare function (SWF)

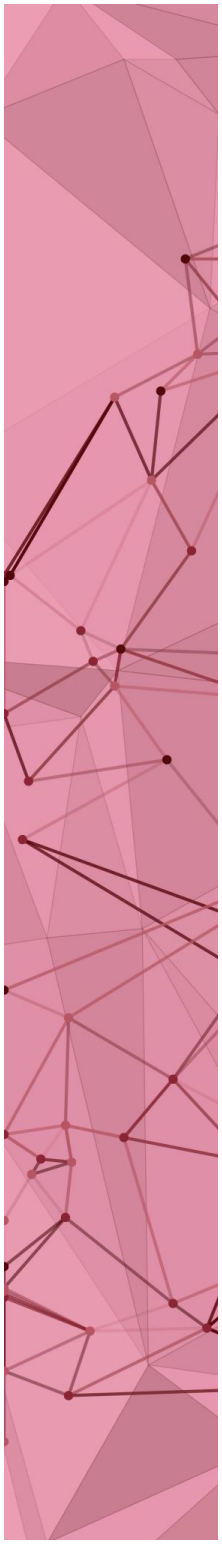
- SWF: Profile → Weak ranking
- SCF vs. SWF
  - Some voting methods don't produce a ranking, just winner(s)
  - SWF may not even exist, e.g., in majority rules (Condorcet paradox)



Condorcet winner

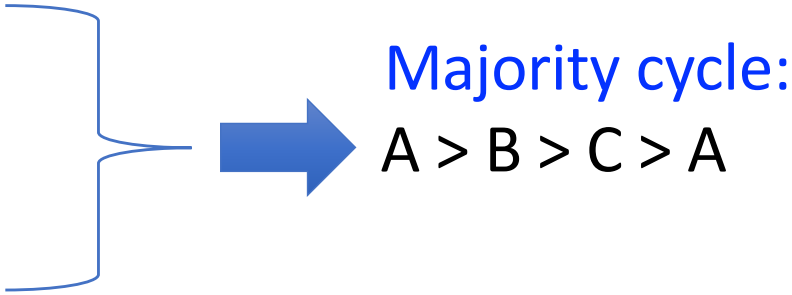
- The candidate that beats every other candidate in head-to-head.

- Example:
- 3 candidates, 3 voters with preferences
- Preferences of the voters:
  - A > B > C
  - B > A > C
  - A > C > B
- Condorcet winner: A
  - A beats B (2-1) and A beats C (3-0)



Condorcet paradox

- 3 candidates, 3 voters with preferences
- Preferences of the voters:
  - A > B > C
  - B > C > A
  - C > A > B

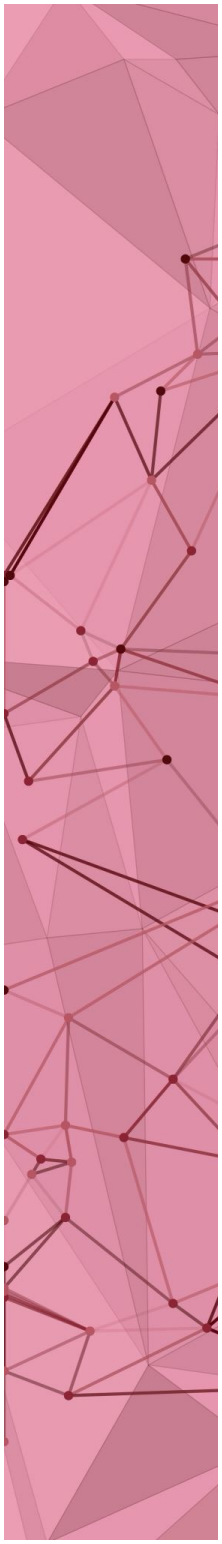


Majority cycle:
A > B > C > A

- A beats B (2-1 vote) and B beats C (2-1)
- But C beats A (2-1)!
- No Condorcet winner

Condorcet paradox

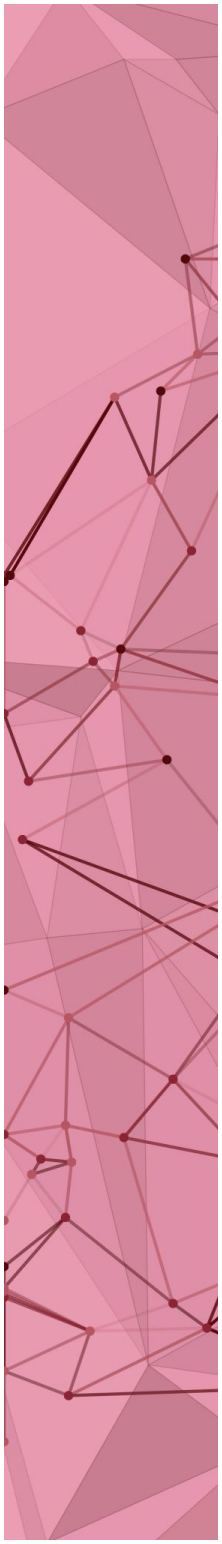
The majority voting outcome may be contradictory (fails transitivity) while individual votes are rational.



Copeland rule

Copeland score of candidate A
= # of candidates that A beats head-to-head –
# of candidates that beat A head-to-head

- Pro: Produces an aggregate ranking by scores (unlike plurality)
- Con: Ignores the margin of head-to-head wins and losses

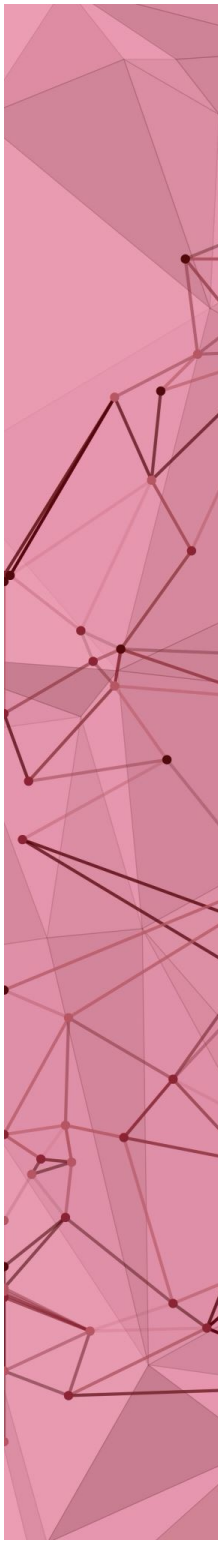


Example: Copeland rule

102 votes	101 votes	100 votes
B	B	C
A	C	A
C	A	B

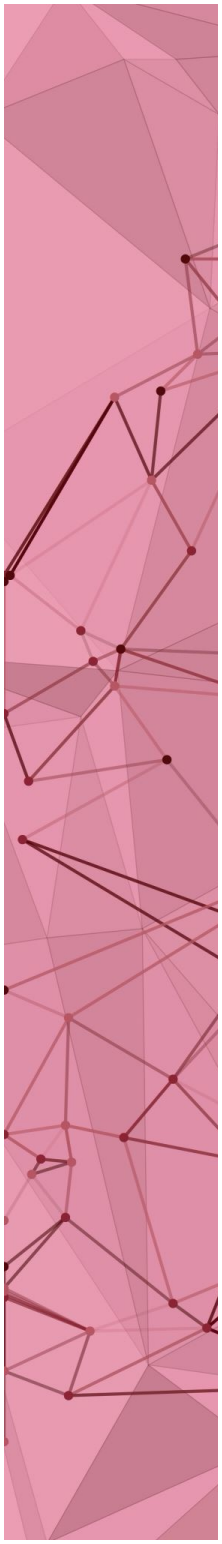
B beats A in 102+101 to 100 votes
C beats A in 101+100 to 102 votes
B beats C in 102+101 to 100 votes

A's score = 0 – 2 = -2
B's score = 2 – 0 = 2
C's score = 1 – 1 = 0
Winner(s) = {B}



More sophisticated method: Borda count

- n candidates
- Each candidate gets n – i points for every voter who ranks them in the i-th place
- The candidate with the most points wins



Example: Borda count

Voter 1	Voter 2	Voter 3
A	B	B
B	C	A
C	A	C

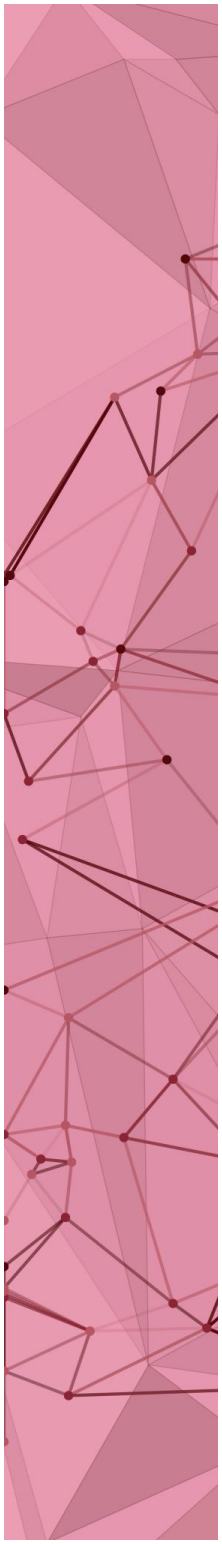
Borda scores:

A: 2 + 0 + 1 = 3

B: 1 + 2 + 2 = 5

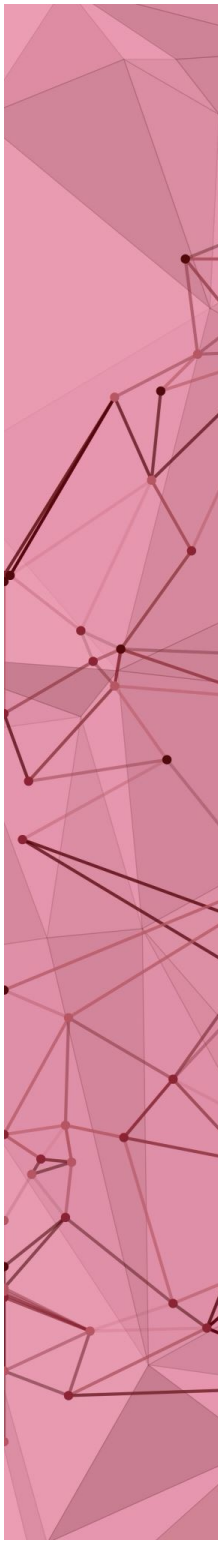
C: 0 + 1 + 0 = 1

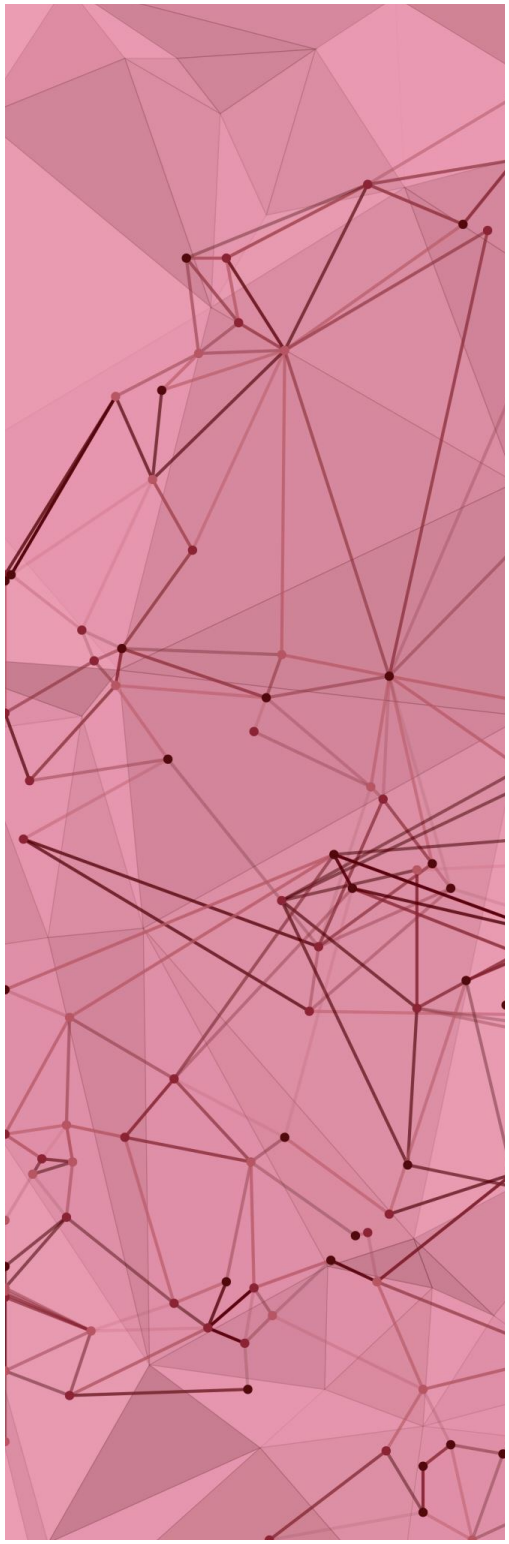
Winner(s) = {B}



Borda manipulation/ strategic or tactical voting

- Jean-Charles de Borda: "My scheme is intended for only honest men."
- Strategic voting
  - A voter's actual preference: A > B > C
  - Knows A will not win
  - Will fake B as his first choice

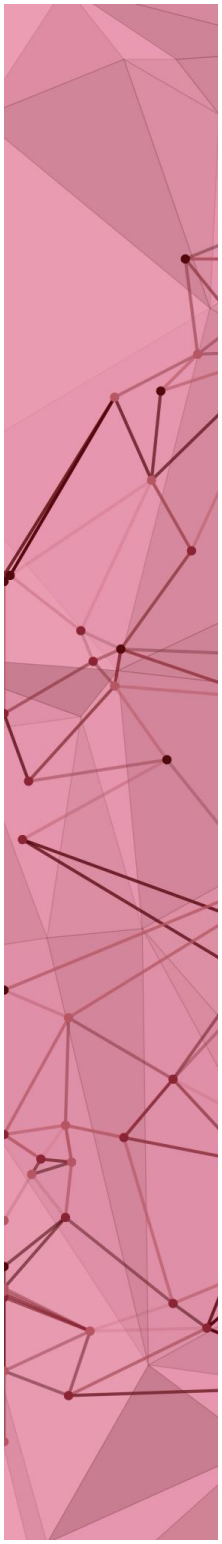




Ranked-Choice Voting (RCV)

Ranked-choice voting (RCV)

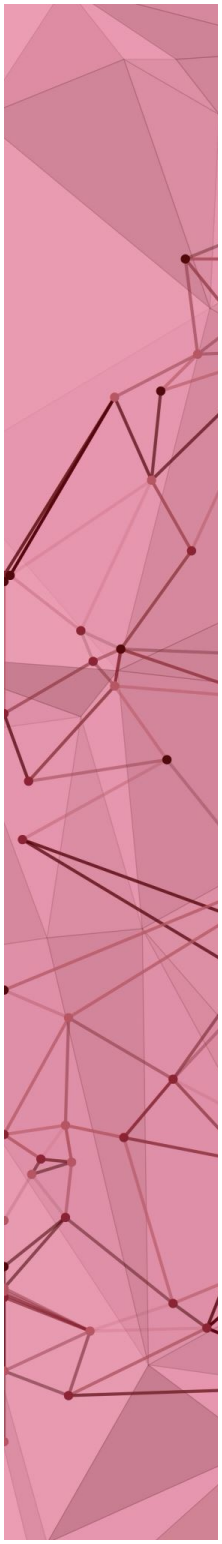
- Multiple rounds
  - Instant run-off voting (IRV): single winner
  - Single transferable voting (STV): multiple winners
  - Many variants: E.g., which scoring rule?



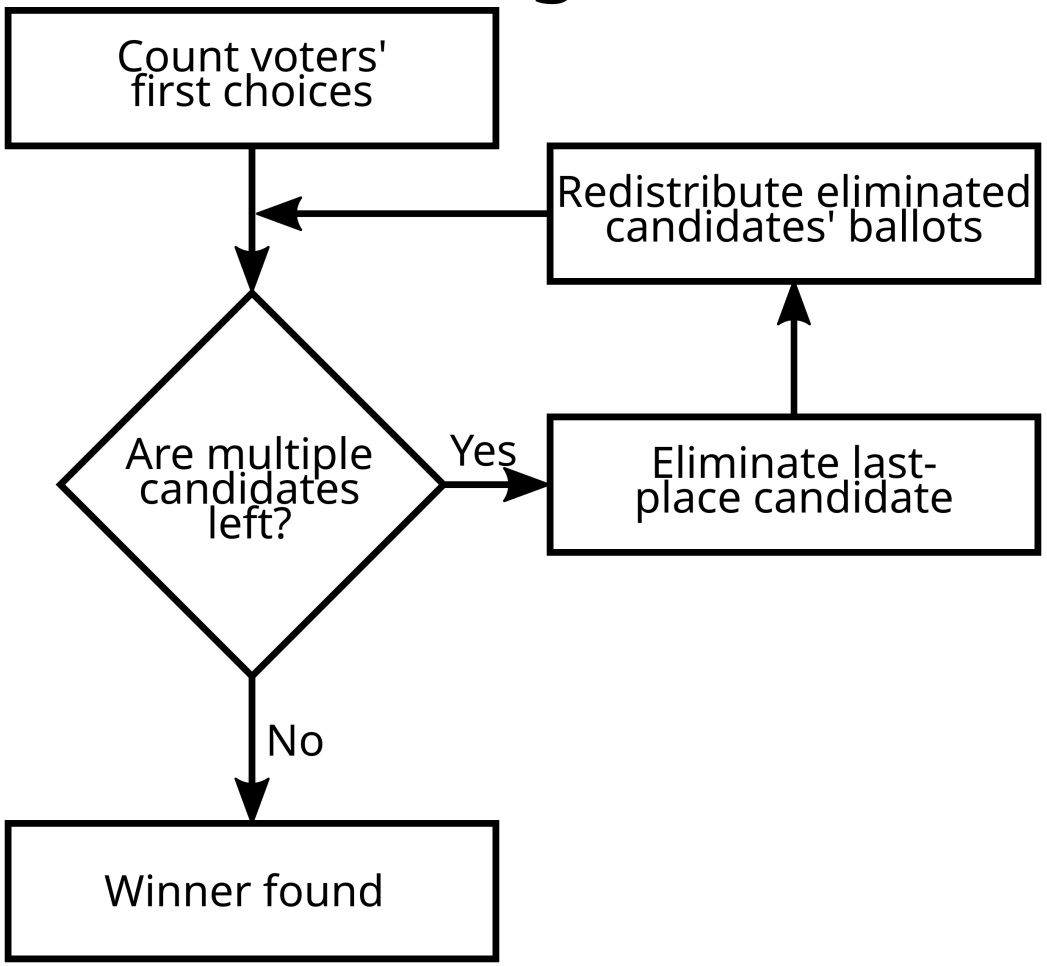
“In the early 2020s, the use of contingent ranked votes saw a comeback in the United States. STV, for a time used only in Cambridge, Massachusetts, was adopted by Portland, Maine, and several other American cities beginning in 2022.

Single-winner ranked voting (specifically instant-runoff voting) is used to elect politicians in the states of Maine and Alaska. In November 2016, the voters of Maine narrowly passed Question 5, approving ranked-choice voting (instant-runoff voting) for all elections. This was first put to use in 2018, marking the inaugural use of ranked votes in a statewide election in the United States.”

https://en.wikipedia.org/wiki/Ranked_voting

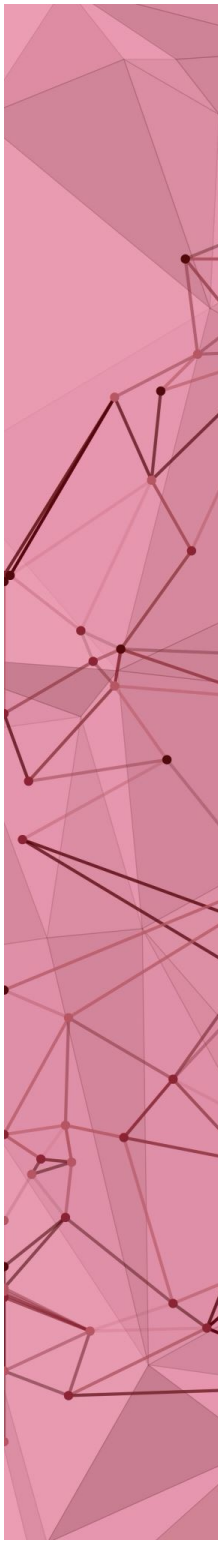


IRV counting flowchart



Example: IRV

https://vote.nyc/RankedChoiceVoting



2025 New York City Democratic mayoral primary

V • E

Candidate	Round 1		Round 2		Round 3	
	Votes	%	Votes	%	Votes	%
Zohran Mamdani	469,642	43.82%	469,755	43.86%	**573,169**	**56.39%**
Andrew Cuomo	387,137	36.12%	387,377	36.17%	443,229	43.61%
Brad Lander	120,634	11.26%	120,707	11.27%	**Eliminated**	
Adrienne Adams	44,192	4.12%	44,359	4.14%	**Eliminated**	
Scott Stringer	17,820	1.66%	17,894	1.67%	**Eliminated**	
Zellnor Myrie	10,593	0.99%	10,648	0.99%	**Eliminated**	
Whitney Tilson	8,443	0.79%	8,525	0.80%	**Eliminated**	
Michael Blake	4,366	0.41%	4,389	0.41%	**Eliminated**	
Jessica Ramos	4,273	0.40%	4,294	0.40%	**Eliminated**	
Paperboy Prince	1,560	0.15%	1,628	0.15%	**Eliminated**	
Selma Bartholomew	1,489	0.14%	1,505	0.14%	**Eliminated**	
Write-ins	1,581	0.15%	**Eliminated**			
Active votes	**1,071,730**	**100.00%**	**1,071,081**	**99.94%**	**1,016,398**	**94.84%**
Exhausted ballots	—		649	0.06%	55,332	5.16%

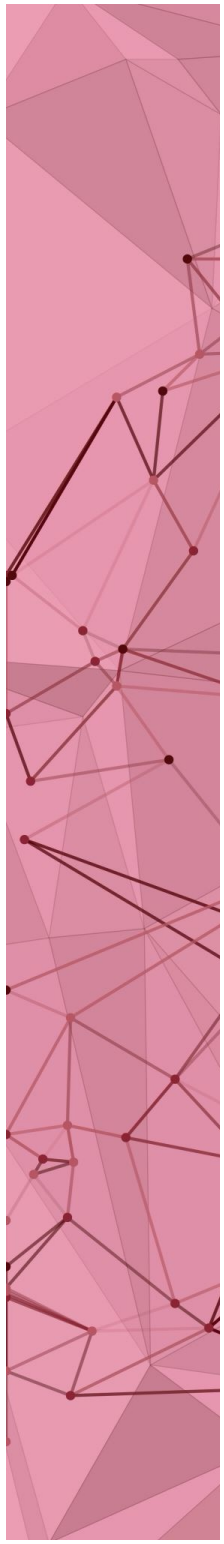
Source: New York City Board of Elections[66]

Non-viable: eliminated at once

Two-Round System (2RS)

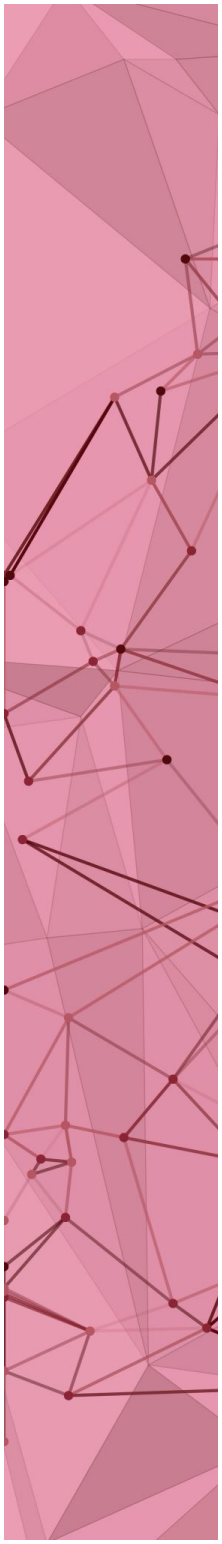
- Origin: France, most common in the world
  - French presidential election
  - US non-partisan primaries in California, Washington, Louisiana, and Georgia
  - Partisan primaries in other states

- First round: Each voter chooses one candidate
  - Top two candidates qualify for the second round
- Second round: Each voter chooses one of the two



Center squeeze

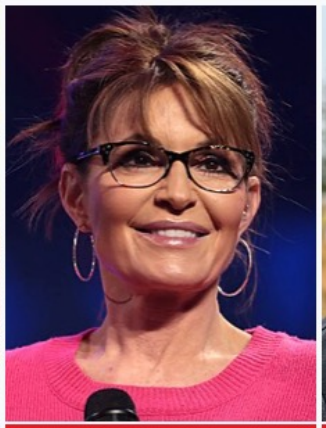
- Known issue with ranked-choice and two-round systems
- Majority preferred candidate (in head-to-head) may lose to extreme alternatives or spoiler candidates (those losing in the end)

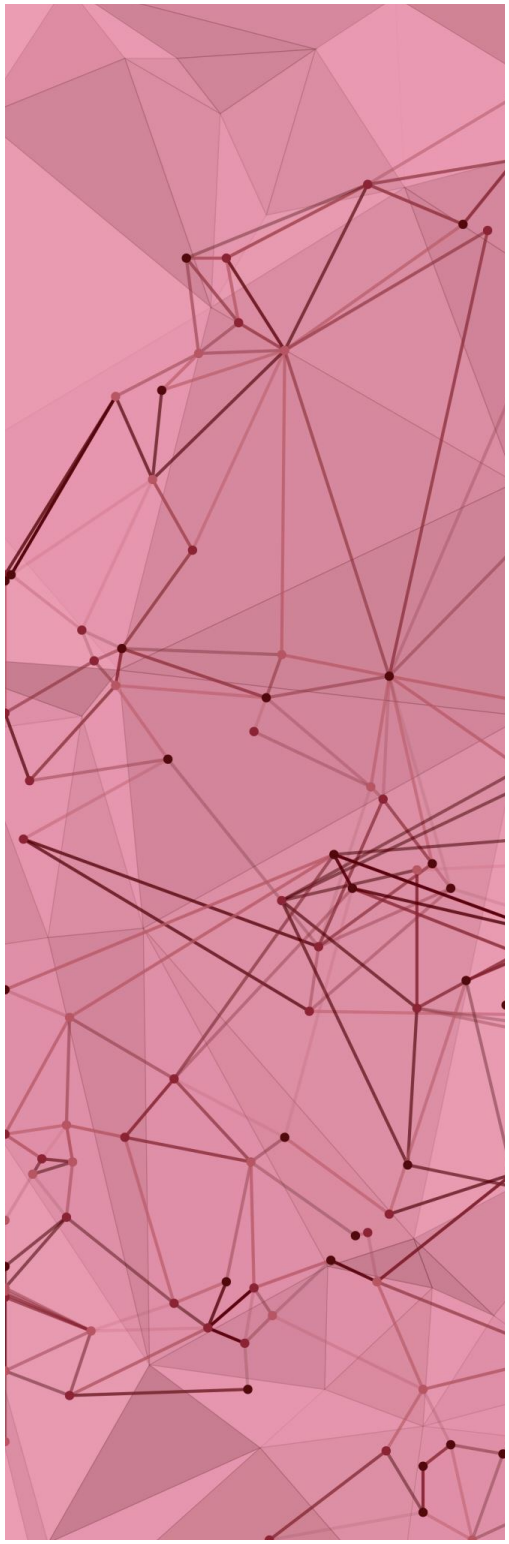


2022 Alaska Special Election:

- Begich is the “Condorcet winner”: wins head-to-head against both Peltola and Palin.
- But he gets eliminated in the first round of IRV.
- Spoiler: Palin

Alaska's at-large congressional district

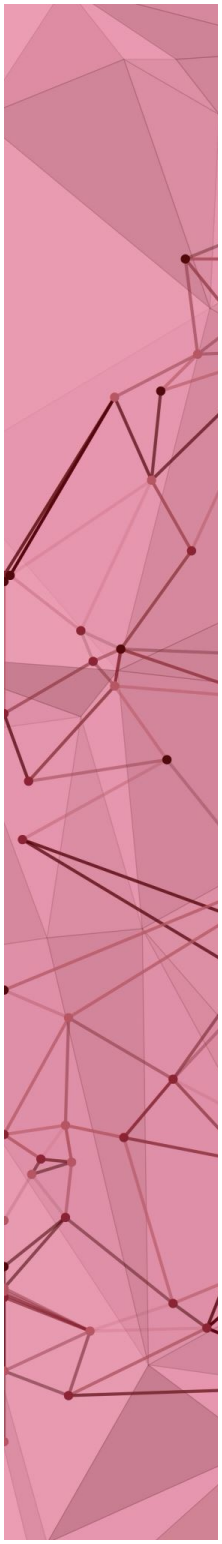
Turnout	32.2%[41]		
			
Candidate	**Mary Peltola**	Sarah Palin	Nick Begich III
Party	Democratic	Republican	Republican
First round	**74,817** **39.7%**	**58,339** **30.9%**	**52,536** **27.8%**
Final round	**91,266** **51.5%**	86,026 48.5%	*Eliminated*



Is there a perfect voting method?

Desirable properties/axioms

1. Unanimity: if all voters prefer A to B, then the social preference must be the same
2. No dictatorship: No one voter dictates the social preference
3. Independent of irrelevant alternatives (IIA): Social preference between A & B depends only on the voters' preferences between A & B, not on other *alternatives* (meaning choices)

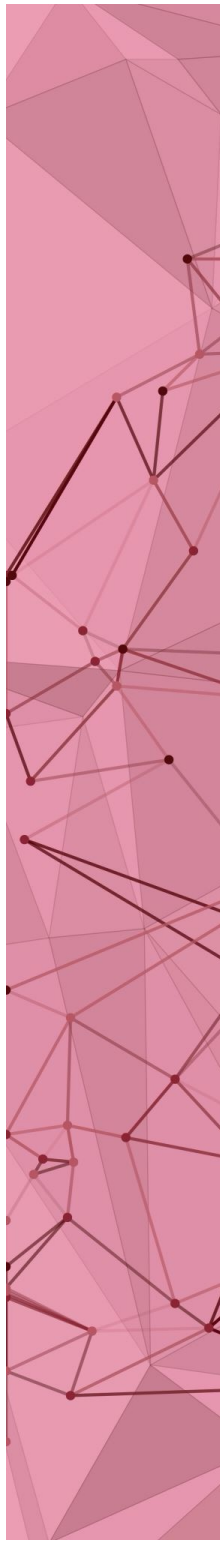


Example: IIA violation

Plurality rule violates IIA

X: X-men, S: Superman, B: Batman

Without X-Men	With X-Men
5: S > B 4: B > S 1: S > B	5: X > S > B 4: B > S > X 1: S > B > X
S > B S beats B (6-4)	X > B > S B beats S (4-1)

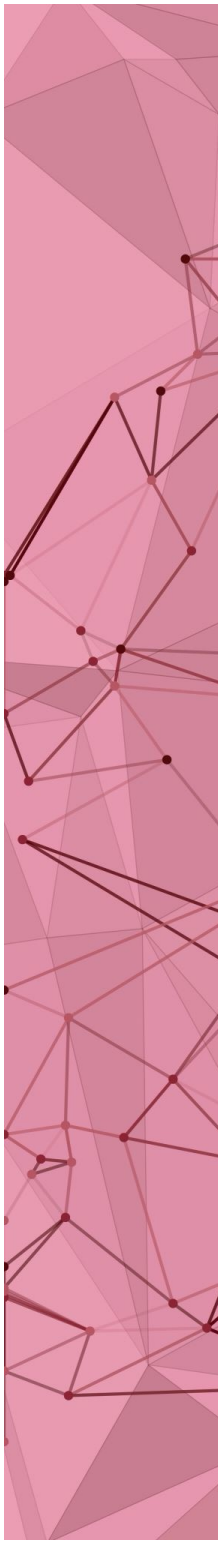


Arrow's impossibility theorem

For 3 or more candidates, there is no voting method that satisfies all 3 desirable properties.

Kenneth Arrow, 1950 (Nobel Prize, 1972)

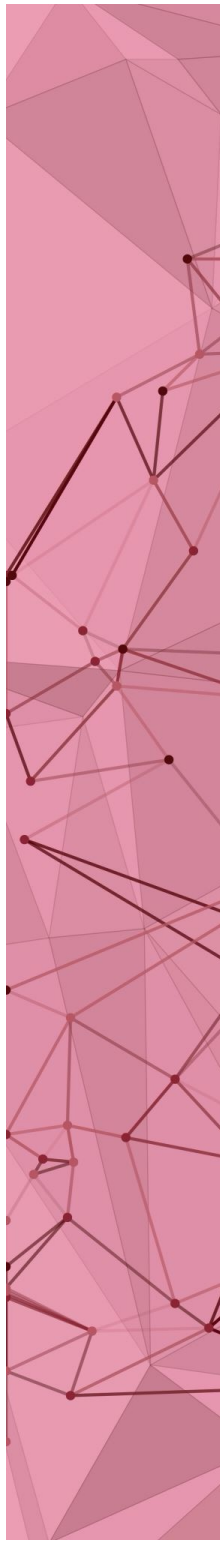
Revived voting theory 150 years after its golden age.



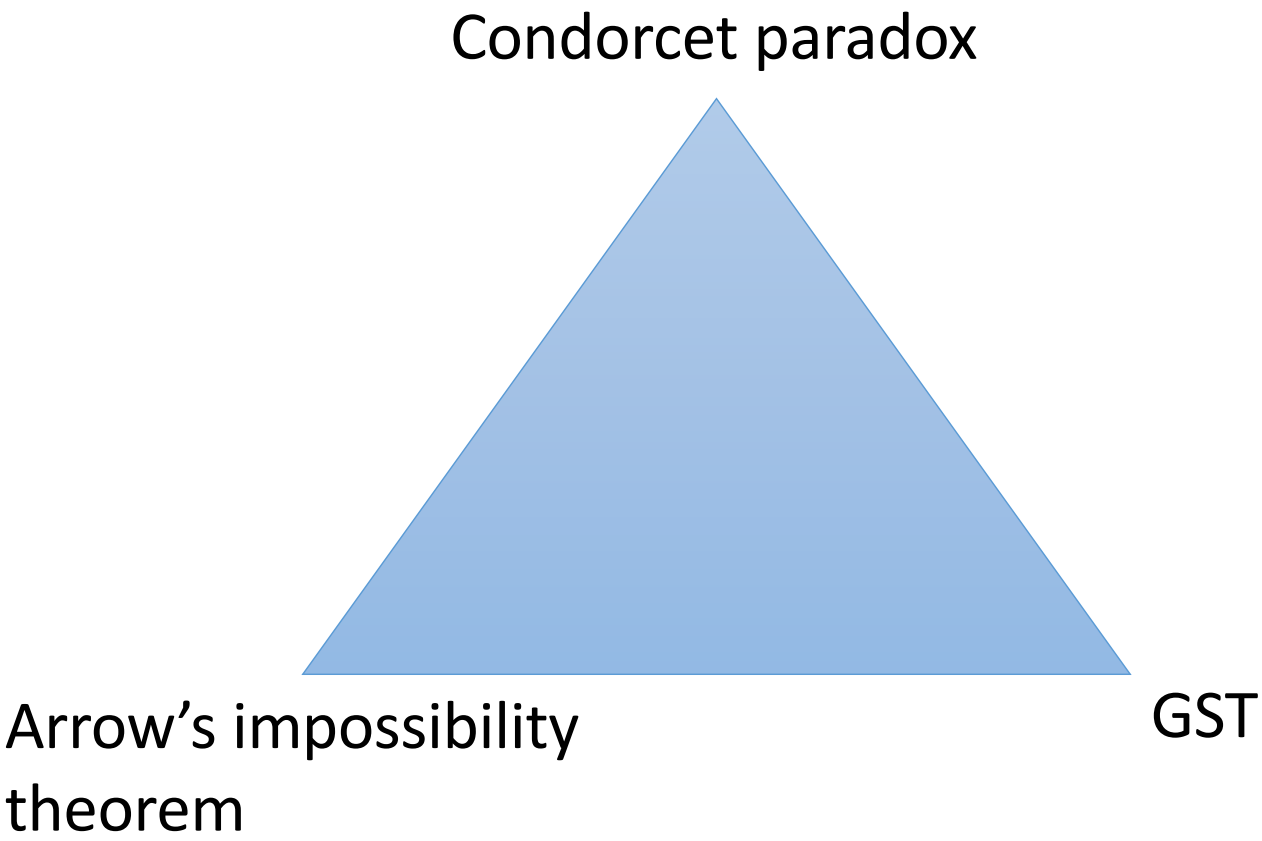
Gibbard-Satterthwaite Theorem (GST)

Other than dictatorship, every ranking-based voting method is susceptible to manipulation (i.e., not *strategyproof*).

Gibbard, 1973 and Satterthwaite, 1975

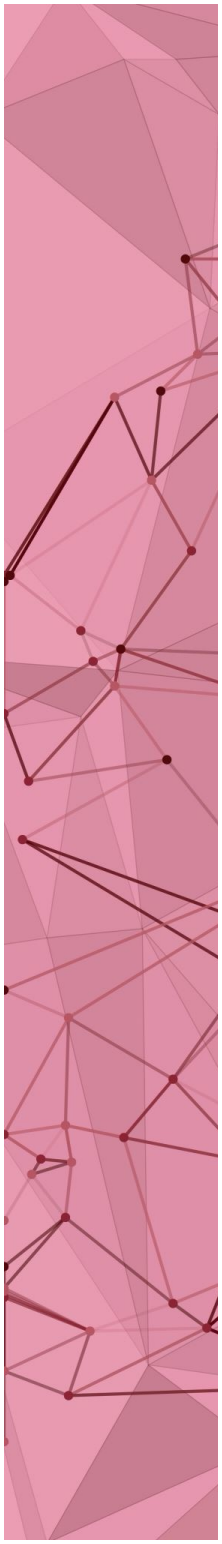


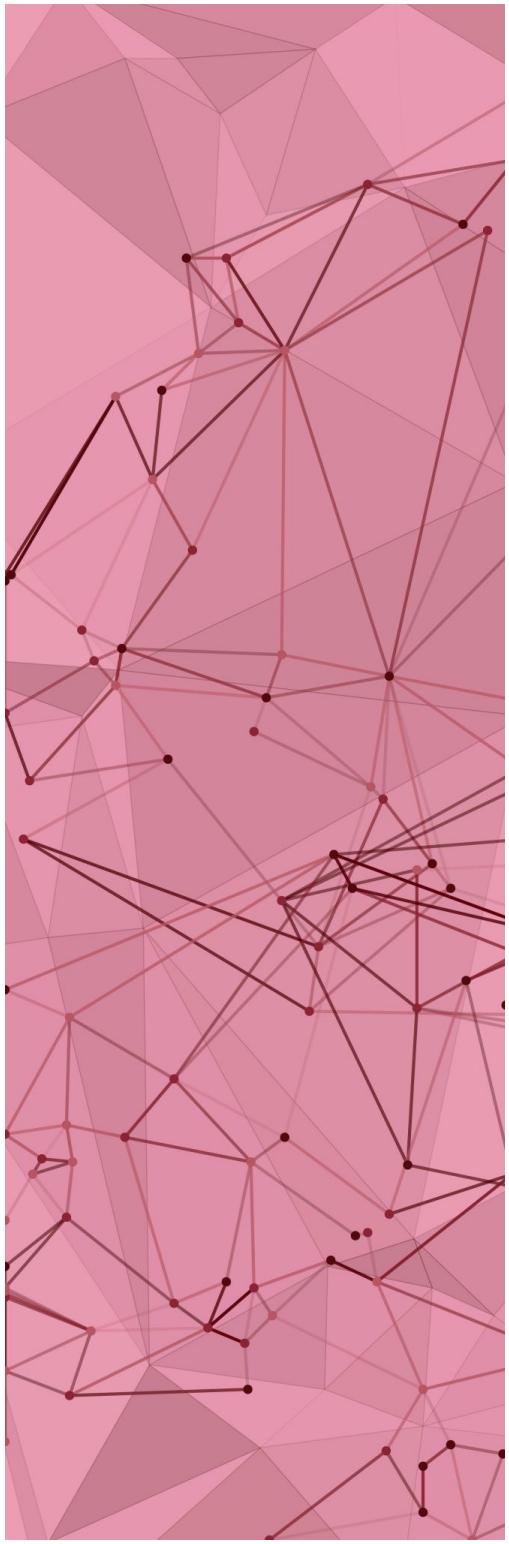
Voting theory triad



Main take-aways

- Avoid love at first sight with any voting method
- Need to critically examine the pros and cons of a voting system
  - Trade-offs among axioms are common
  - “Fair voting system”: how and at what cost?
- Strategyproof voting is important, but how hard is manipulation computationally?





Axiomatic approach to evaluating voting methods

Axioms: Precisely defined properties of voting rules, often normative

Axioms I

Bare minimum

Absolutely required

Axioms II

Higher demands

Tradeoffs unavoidable

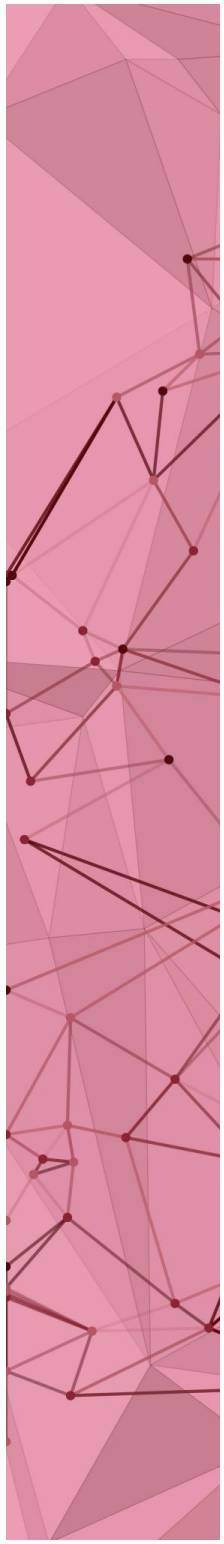
Controversial

Axioms III

Strongest demands:

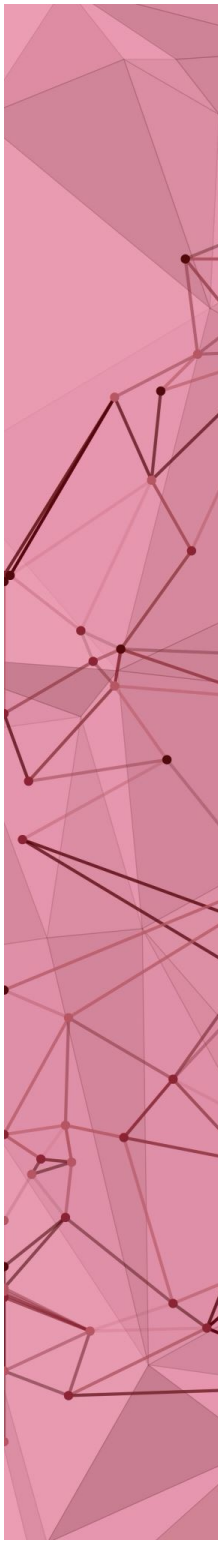
strategyproofness, IIA

Impossibility thms apply



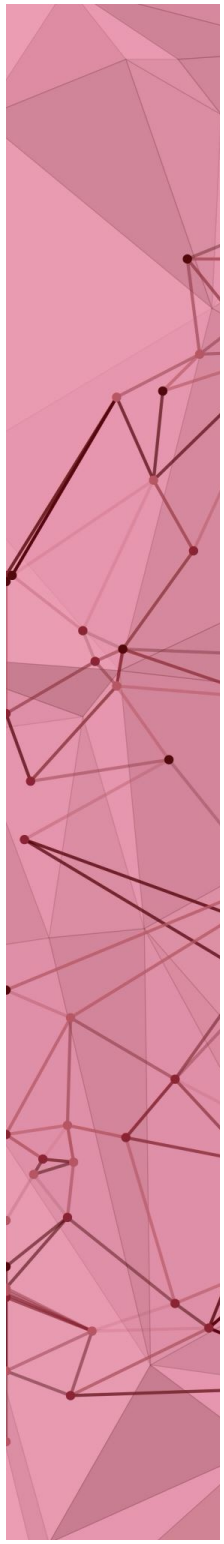
Axioms I

- Anonymous: Voters are treated the same: no voter has more weight than another
- Neutral: All candidates are treated equally: swapping two candidates in everybody's ballot results in swapping them in the outcome
- Pareto optimality: The voting method never declares X as a winner when every voter prefers some other candidate to X



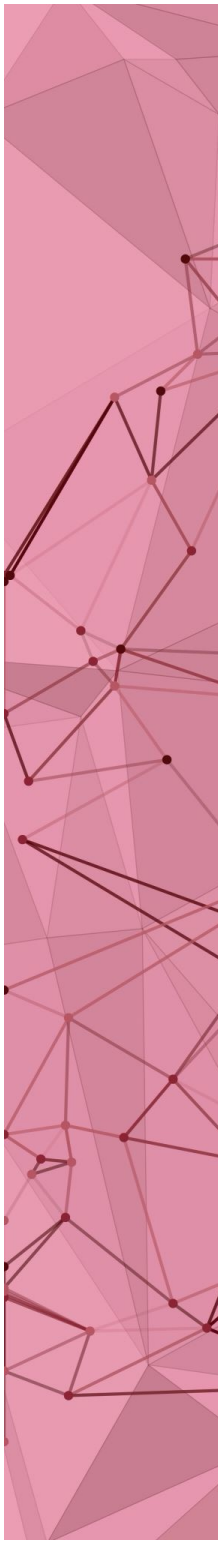
Which methods satisfy Axioms I?

- Plurality
- Copeland
- Borda
- Numerous variants
- Multi-round versions of these: IRV, STV, etc.



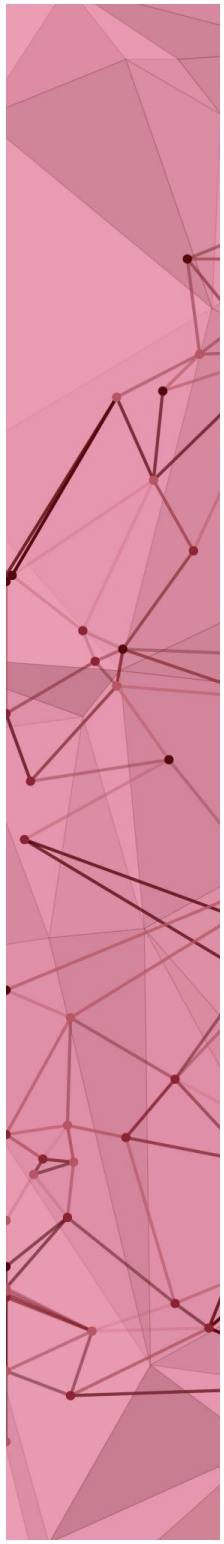
Axioms II

- Reinforcement:
  - Let's say candidate X wins in Brunswick and Portland separately
  - Combine the votes of Brunswick and Portland
    => X should still be the winner
- Sampling of results:
  - Condorcet consistent methods (which declare Condorcet winner if there exists one) don't satisfy reinforcement for 3 or more candidates.



Axioms III

- Strategyproofness
- IIA
- Impossibility theorems
  - Can debate the importance of IIA in democracy

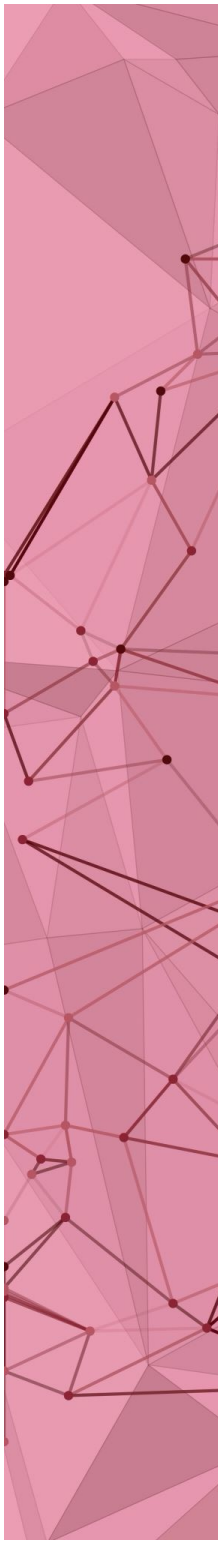


Approval Voting

Vote for as many candidates as you like.

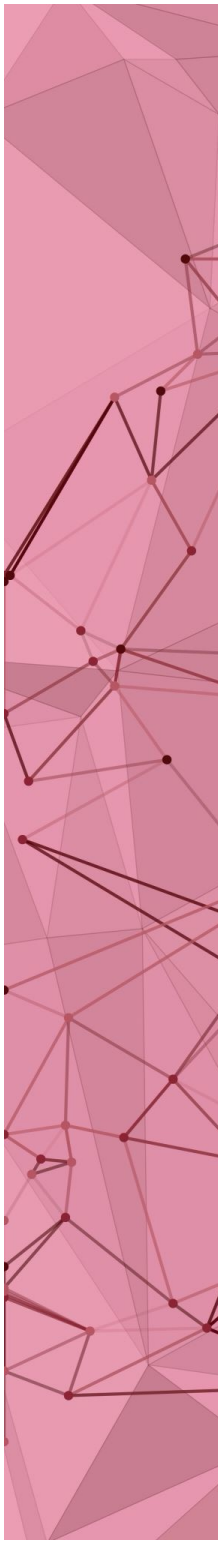
Andre	○
Blake	●
Carmen	●
David	●
Ella	○

The candidate with the most votes wins.



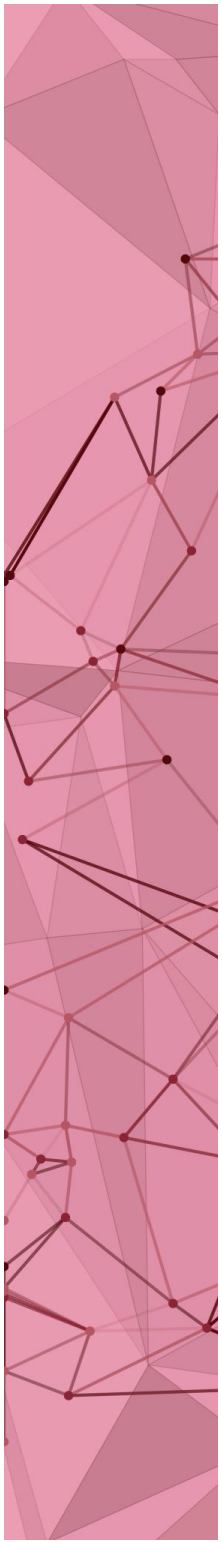
Approval voting: SCF

- Does not have an SWF or SCF: ballots are not ranked
- Construct an SCF
  - Two indifference classes for a ballot: marked & unmarked
  - Indifference among all marked candidates
  - Marked candidates are preferred over unmarked
- We can now apply Borda and other voting rules



Pros: approval voting

- Simplicity
- Addresses the #1 flaw of plurality voting: vote splitting between same-party candidates
- Improves the probability that the winner will be supported by the majority of citizens
- Relatively resistant to strategic manipulation



Cons: approval voting

- Understanding the meaning: where to draw the line of approval?
- Restricts the expressiveness of a ballot
- Violates “one person, one vote”
- Unfair: gives more influence to voters who approve more candidates